Phil 325

Logic II

Time & Place: MW 2.45-4.00, Lawrence 220
Instructor: Nate Bulthuis
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Office Hours: T 5-6, W 2.30-4.30
Office: Hascall 206

Course Description

Logic I developed and applied systems of proof. In this more advanced course, we are concerned with the proof systems themselves, and especially with the question of what can and cannot be proven from a given set of premises. This question has an intimate connection to the question of what can be computed by a digital computer, and we will start out with a brief overview of the theory of Turing computability. After that, we prove three main results of meta-logic: Alonzo Church's theorem that first-order logic is undecidable, the Löwenheim–Skolem theorem that every consistent set of sentences has an at most countably infinite model, and Kurt Gödel's completeness theorem for first-order logic. The remainder of the course is devoted to proving what is probably the most important (and most surprising) result in the history of logic: Gödel's incompleteness theorems for arithmetic.

Prerequisites

Logic I or permission of the instructor

Reading

You need to purchase the following textbook for this course, available at the Colgate bookstore:

Boolos, G, J. Burgess and R. Jeffrey, *Computability and Logic,* fifth edition. Cambridge University Press, 2007.

Be sure to purchase the **fifth edition**.

Grading

Ten Problem Sets (40 pts/set): Problem Sets are due at the end of the week each Sunday.

Mid-Term Exam (250 pts): A mid-term focusing on the first two units of the course

Final Exam (300 pts): A two-hour cumulative exam, with a focus on the third unit of the course.

Date	Notes	Reading	Topics
Week 1 8.27			**0. Introduction**
Week 2 8.31 9.2		 Chapter 1 Chapter 2	**1. Computability** Cantor's Theorem
Week 3 9.7 9.9		 Chapter 3 Chapter 3	 Turing Machines
Week 4 9.14 9.16		 Chapter 4 Chapter 4	 Turing Machines
Week 5 9.21 9.23		 Chapter 9 Chapter 10	**2. Basic Metalogic** First-Order Logic
Week 6 9.28 9.30		 Chapter 11, Sec. 1 Chapter 12	 Church's Theorem Löwenheim-Skolem Theorem
Week 7 10.5 10.7		 Chapter 12 Chapter 14	 Löwenheim-Skolem Theorem Gödel's Incompleteness Theorem
Week 8 10.12 10.14	Fall Break	 Chapter 14	 Gödel's Incompleteness Theorem
Week 9 10.19 10.21	Mid-Term	 Chapter 6	**3. Gödel's Theorems** Recursive Functions
Week 10 10.26 10.28		 Chapter 6 Chapter 7	 Recursive Functions
Week 11 11.2 11.4		 Chapter 7 Chapter 11, Sec. 2	 Recursive Functions
Week 11 11.9 11.11		 Chapter 15 Chapter 15	 Gödel's Numbering
Week 12 11.16 11.18		 Chapter 16 Chapter 16	 Gödel's Numbering
Week 14 11.30 12.2		 Chapter 17 Chapter 17	 First Incompleteness Theorem
Week 15 12.7 12.9		 Chapter 18 Chapter 18	 Second Incompleteness Theorem